REPORT OF THE COMMITTEE ON ELEMENTARY-SCHOOL MATHEMATICS, OF THE ASSOCIATION OF MATHEMATICS TEACHERS OF THE MIDDLE STATES AND MARYLAND.

Your Committee on Elementary-School Mathematics is again under the painful necessity of dashing to earth any hopes and expectations which may have arisen from the announcement in the program looking forward to a final report.

If there is any one thing which the work of this committee lacks, it is finality. The best we can do at this time is to present for your consideration certain tentative recommendations in the hope that still further light may be shed upon our difficult problem, and that the members of the Society may devote a modicum of time and attention to its consideration.

Let it be stated at once that we are not prepared to lay down a definite program or set forth an ideal curriculum. Our investigations have brought to light many widely varying opinions which were dependent on local conditions in general and upon administrative questions in particular. We feel that a syllabus, such as we might outline at this time, would be highly artificial and nowhere generally acceptable.

The trend of mathematical pedagogics for the past decade has taken two directions which, at first thought, seem contrary: these are (1) a demand for greater rigidity and clarity in definition and development of the fundamental processes. (2) A search for more numerous points of contact between the world of logic and the world of experience. The immediate result has been an apparent division of our forces into opposing camps. It should be noted at once that there is no really essential contradiction involved in these two distinct aims; provided only that the antique doctrine of immediate intuition of fundamentals is abandoned; a question which it is not within our province to consider. But what *is* inescapably involved is a careful and prolonged study on the part of teachers as to the relative degree of refinement of process suitable for classroom purposes under

varying conditions, and the relative extent to which, in our teaching, application should be made of mathematical processes to actual concrete facts. This matter is one which it is well nigh impossible to dogmatize about. The best solution seems to lie in the direction of a minimum syllabus of essentials, allowing the widest liberty of choice of applications, depending upon local conditions; and also giving due regard to individual and local differences in degree of recognition of the demands of logical accuracy. In reference to the last mentioned, it may well appear that the steps, in each demonstration or solution, form a *continuous* class, which is simply isomorphic with a concrete instance, and that no *proof* is really perfect as stated. Even the Euclidean processes of logic, strict as they are, are capable of further intensive development.

The moral of which is that we are not justified in expecting from our students either in elementary school or in high school that they attain a fixed standard of logical precision, Euclidean in geometry, or Robinsonian in arithmetic. It is the fear of violation of Euclidean (or Wentworthian) standards of process which has kept geometry out of the elementary schools. Teachers have failed to distinguish between the significance of error in conclusion and that of incompleteness in process. Both are regarded as delinquencies and equally culpable. At the outset, then, we may safely assert that *degrees of accuracy* and *degrees of completeness* should be recognized and carefully distinguished by teacher and pupil alike.

To illustrate: It should appear that the results of a financial transaction must be accurate to the last cent, but no further; but that to find correct to a ten thousandths of a square inch the amount of tin needed for a cylindrical bucket is highly absurd. So it is absurd to expect from a fourth-grade pupil any form of "proof" of the rule for division of fractions, or of a high-school pupil (except possibly in the fourth year) a "proof" by the so-called method of limits; but it is equally a source of discord between elementary-school and high-school mathematics for the elementary-school student to escape entirely the demand for fairly careful abstract reasoning.

The tendency in elementary schools for the past two decades seems to have been away from all abstractions. In their effort

to make the arithmetic *applicable,* the elementary-school authorities seem to have limited the arithmetic to consideration of concrete magnitude. This doubles the difficulty encountered by the student upon entering high school. In most cases, not only must he master a new notation, but he must also comprehend the significance of general truths in mathematics. What teacher has not found the statement of a troublesome principle made perfectly clear by the insertion of the word "dollars" at certain convenient points? Any abstract principle is, by its very nature, removed from the particular concrete setting which gave it birth. It requires to be memorized carefully, and its general character emphasized by a large variety of illustrations. This process must begin early, and should never be lost sight of during the student's entire mathematical experience.

There has been unquestionably a decline in the quality and scope of mathematical instruction in the elementary schools during the past quarter century. It is not our purpose to lay the blame for this, if such it is, upon any one in particular. The question is lost in the more general problem of overcrowded curricula. There is little doubt that the arithmetic of our fathers has been elbowed out by the study of history and English. But this may have been necessary. Under existing conditions it is of doubtful value to urge that a very large share of the child's time and attention be given to mathematics in the elementary school. And just here we encounter a difficulty which makes it impossible to draw final conclusions at this time.

Other elementary-school studies, particularly English, are concerned chiefly with the student's immediate environment. Mathematics involves training in a field which is largely foreign to his daily experience. To just this extent it is *forced* upon him. We should not, of course, lose sight of the appeal which manipulation of number symbols and other symbols makes to the imagination. But this is admittedly small in comparison with the demands of physical necessity, or the desire for self-expression. We teachers of mathematics are looking to the future of our pupils. Our science and art are primarily instrumentalities for which they do not instinctively recognize a need. This places our subjects in the category called "disciplinary," assuming this term to signify "directed toward an end not immediately present

to the consciousness of the pupil." Twenty-five years ago all schooling was largely disciplinary in this sense. At present much of it is anything but that. Ideals and modes of discipline are out of fashion (or were until eight months or so ago). It is unnecessary to justify this statement. One need only to state that while the aim of modern educational method is self-expressive, the ideal of discipline is self-repressive.

The natural result of this change of ideal was, as we all recall, a perfect orgy of inaccuracy and carelessness. Self-expression reached the point where much was expressed that might better have been left concealed. The results are seen to-day, not only in our own particular field, but in the daily press and in certain prevailing modes of fashion. "Educators" who had previously bombarded the teaching forces with "methods of development" and "inductive plans" were not slow to sense this danger. But their sole remedy seems thus far to have been found in incessant drill in the fundamental operations of numbers. This has reached the point where tests in computation are standardized, and progress is determined by these standards. If the whole of the arithmetic of the earlier generation were covered in this fashion, the process would require fifteen or twenty years, at least. So naturally the next stage in this interesting evolution is reached by a most extended list of omissions. In a recently issued state report, it was recommended that, except for decimals, the only fractions taught in the elementary school should be halves, fourths, eighths and sixteenths, with thirty-seconds for the brighter pupils. In many schools square root, proportion, greatest common divisor, lowest common multiple, numerical factors, circular measure and the fundamental principles of multiplication, division and fractions are not taught at all! Small wonder that in high school we find it difficult to extend these topics into the field of negative number and literal expression.

The remedy for existing conditions is not easy to find. It cannot be reduced to a few well-chosen sentences. But it probably lies along certain lines of action which at least are not mutually inconsistent.

(*a*) Accuracy should be sought not by endless repetition, but by a return in some degree to disciplinary methods in beginning arithmetic.

(*b*) Immediate interest as affording the teacher a mode of attack must to some extent give way to emphasis upon the importance of general principles.

(*c*) Percentage and its applications should be dethroned as the be-all and end-all of the higher elementary grades. The fact of its widespread use in commercial transactions should not overshadow its comparatively minor mathematical significance, but should be considered as one aspect of the problem of vocational training. Incidentally it may be noted that such training, to be a success, can hardly be expected to end with the eighth school year. If anywhere short of a complete high-school course, it can hardly be completed within a total of ten years. This leaves time for a thorough course in *Commercial Arithmetic* in the ninth year for those whose vocational choice is commerce, rather than, say, housekeeping, or aviation.

(*d*) Between the elementary arithmetic and the orthodox course in secondary algebra there should be given a distinct course in mathematics. The question of the make-up of this course might well be referred to a committee with instructions to report in six years. Let us devote a few minutes to the consideration of the present status of this part of the main problem.

We may as well understand at the outset that certain geometrical facts and certain algebraic relations of quantity are of as much importance mathematically, psychologically and practically, as is trade discount to the third remove, or the ultimate partiality of partial payments. Of these, sequence, direction, symmetry, congruence, similarity, area, average and transformation in general may be specified. There is also the important concept of definite magnitude independent of its numerical measure—of such a character that the measure depends equally upon the magnitude and upon the unit. These concepts are for the most part left without explicit enunciation until the conclusion of the high-school course, or omitted entirely. They seem to belong in the elementary course. Here they should be based neither upon purely empirical nor purely rational processes, but upon a wise combination of the two, in glorious disregard of Euclidean type forms.

The question of negative number demands our attention. It may well be that our problem would be greatly simplified by a

postponement of this topic to the period of formal algebra. Apparently much of our difficulty lies here. We have from the outset employed the *mechanism* of negative number in our algebraic processes to the exclusion of its *significance*. In so doing we have given the algebra an artificial character which it ought not possess. This concept is difficult enough in its simplest forms. In the form in which it appears, it passes entirely beyond the powers of realization of the student. It may well be that it stands in the way of his appreciation of the meaning of other equally important mathematical principles. The Greeks, to say the least, got on very well without it.

In such a course, the *principles* of arithmetic may well be brought into clear relief. At the same time the value of literal and other algebraic symbolism may be set forth with vigor and effect. Emphasis would naturally be placed upon the derivation and evaluation of formulæ, and ultimately upon various transformations of formulæ.

The problem of relative value of respective fields of application demands exhaustive study. In the past, as has been said, methods of commerce and exchange have displaced other considerations in the elementary curriculum. All admit the need of a change in this regard—but to what? The very difficult question of differentiating groups of pupils along vocational lines is encountered here. So many administrative problems are involved that it seems impracticable to offer definite recommendations at this time. The synthesis of applied problems must depend upon group analysis of detailed demands of the various vocations. It is possible, however, that a mathematical study of these demands may be of value in the work of vocational guidance.

At the time of the appointment of this committee, reference was made to the curriculum of the junior high school. The organization of these schools is primarily an administrative question. It would be most unfortunate if the reform of elementary-school mathematics were postponed until these schools shall be organized throughout the country. There is no essential reason for discriminating between the work of a teacher with a class in a junior high school, and the work of the same teacher teaching the same class at the same school age in an elementary school;

nor is there any apparent reason why the work of the seventh and eighth elementary grades should not, under existing conditions, be to some extent departmentalized. So much has been said in favor of the organization of the seventh, eighth and ninth years into an administrative unit that endorsement of the junior high-school plan would be superfluous at this time. It is sufficient that emphasis be placed upon the maintenance of continuity throughout the mathematics of the entire school course.

In conclusion, we would respectfully submit to our friends who are psychologically minded the desirability of a study of progressive standards of intricacy of abstract process, and of the power of generalization, as well as of accuracy.

William James has set the aim of philosophy as the ability to know a good man when one sees him. It may well be *one* aim of a properly ordered mathematical course to know a mathematical principle when one sees it. Is it too much to hope that a sound mathematical training may aid our boys and girls in recognizing a *moral* principle when they see it?

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